KOLEJ YAYASAN PELAJARAN JOHOR
FINAL EXAMINATION

COURSE NAME	:	**ENGINEERING MATHEMATICS 3**
COURSE CODE	:	**MAT2033**
EXAMINATION	:	**MAY 2017**
DURATION	:	**3 HOURS**

INSTRUCTION TO CANDIDATES

1. Answer **ALL** Question in the answer booklet

2. Candidates are not allowed to bring any material to examination room except with the permission from the invigilator.

3. Please check to make sure that this examination pack consist of:
   i. Question Paper
   ii. Answer Booklet

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

*This examination paper consists of **8** printed pages including front page*

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET

JAWAB SEMUA SOALAN DI DALAM BUKU JAWAPAN

QUESTION 1

a) Find $\frac{dy}{dx}$ of the function:

Dapatkan $\frac{dy}{dx}$ *bagi fungsi:*

$$y = \frac{x^2 - 2x + 1}{x^3}$$

[3 Marks]

b) Integrate the function by using substitution method:

Kamirkan fungsi dengan menggunakan kaedah gantian:

$$\int (5x-2)^3 \, dx$$

[3 Marks]

QUESTION 2

Use the method of separation of variables to solve the equation:

Guna kaedah pemisahan pembolehubah untuk menyelesaikan persamaan:

$$y^2 \frac{dy}{dx} = \frac{3x + x^3}{y}$$

[4 Marks]

QUESTION 3

Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:

Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y'' - y' - 6y = 9x^2 + 4$$

[7 Marks]

QUESTION 4

a) Evaluate:

Nilaikan:

$$L\left\{2t^5 + 3e^{-t} - \sin 3t\right\}$$

[3 Marks]

b) Use the method of Laplace transforms to solve the initial value problem:

Gunakan kaedah jelmaan Laplace untuk menyelesaikan masalah nilai awal:

$$y'' + 5y' + 4y = 0, \qquad y(0) = 1, \qquad y'(0) = 0$$

[8 Marks]

QUESTION 5

a) Find the partial derivative f_x , f_y , f_{xx} and f_{yx} :

Dapatkan terbitan separa f_x , f_y , f_{xx} *dan* f_{yx} *:*

$$f(x,y) = x^3y^2 + x^2 - \cos y$$

[4 Marks]

b) If $f(x,y) = e^y + e^x$, where $x = u\cos v$ and $y = v\sin u$, use the chain rule to find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$.

Jika $f(x,y) = e^y + e^x$ *, dengan* $x = u\cos v$ *dan* $y = v\sin u$ *, gunakan aturan rantaian untuk mendapatkan* $\frac{\partial f}{\partial u}$ *dan* $\frac{\partial f}{\partial v}$.

[5 Marks]

c) Find the maximum, minimum and saddle points of the function:

Dapatkan titik maksimum, titik minimum dan titik pelana bagi fungsi:

$$f(x,y) = x^2 + 3y^2 - 2x - 3y + 11$$

[7 Marks]

QUESTION 6

a) Evaluate the double integral:

Nilaikan kamiran ganda dua:

$$\iint_R xy^2 \, dA \text{ where } R = \{(x,y) \mid 1 \le x \le 3, \quad 0 \le y \le 2\}$$

[6 Marks]

b) Sketch the region of integration and reverse the order of integration:

Lakarkan rantau kamiran dan tukarkan tertib kamiran:

$$\int_{-3}^{0}\int_{0}^{x^2} f(x,y)\, dydx$$

[5 Marks]

c) Use polar coordinates to evaluate the integral $\iint_R \left(x^2+y^2\right) dA$ where R is the region enclosed in the circle $x^2+y^2=4$

Gunakan kamiran kutub untuk menilaikan $\iint_R \left(x^2+y^2\right) dA$, *dengan* R *ialah rantau tertutup dalam bulatan* $x^2+y^2=4$

[5 Marks]

END OF QUESTION PAPER

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Hyperbolic Identities

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\cosh^2 x - 1$

$\cosh 2x = 1 + 2\sinh^2 x$

Derivatives Formulas

First Principle: $f'(x) = \lim_{\partial x \to 0} \dfrac{f(x+\partial x) - f(x)}{\partial x}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}x^n = nx^{n-1}$

$\dfrac{d}{dx}e^x = e^x$

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$

$\dfrac{d}{dx}\sec x = \sec x \tan x$

$\dfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\dfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x\ln a}$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$

$\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c = Ae^{m_1 x} + Be^{m_2 x}$
2. real and equal: $m_1 = m_2$	$y_c = Ae^{mx} + Bxe^{mx}$
3. complex numbers: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1, m_2	y_p
$A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$m_1 \neq 0$ and $m_2 \neq 0$	$B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0$
	$m_1 = 0$ or $m_2 = 0$	$(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)x$
$Ke^{\alpha x}$	$m_1 \neq \alpha$ and $m_2 \neq \alpha$	$Be^{\alpha x}$
	$m_1 = \alpha$ or $m_2 = \alpha$	$Bxe^{\alpha x}$
	$m_1 = \alpha$ and $m_2 = \alpha$	$Bx^2 e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1 \neq \beta i$ and $m_2 \neq \beta i$	$B_1\cos\beta x + B_2\sin\beta x$
	$m_1 = \beta i$ or $m_2 = \beta i$	$(B_1\cos\beta x + B_2\sin\beta x)x$

Table of Laplace Transform $L\{f(t)\}=F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at}f(t)$	$F(s-a)$
8	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at}\cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at}\cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t\sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t\cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n=1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y'(t)$	$sY(s)-y(0)$ with $Y(s)=L\{y\}$
19	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$