KOLEJ YAYASAN PELAJARAN JOHOR
ONLINE FINAL EXAMINATION

COURSE NAME	**:**	**ENGINEERING MATHEMATICS III**
COURSE CODE	**:**	**MAT 2033**
SESSION	**:**	**DECEMBER 2021**
DURATION	**:**	**3 HOURS**

INSTRUCTION TO CANDIDATES /
ARAHAN KEPADA CALON

1. This examination paper consists of **ONE (1)** part : / PART A (60 Marks) /
   *Kertas soalan ini mengandungi **SATU (1)** bahagian:* *BAHAGIAN A (60 Markah)*

2. Answer ALL questions in the answer sheet which is A4 size paper (or other paper with the consent of the relevant lecturer). /
   Jawab SEMUA soalan di dalam kertas jawapan iaitu kertas bersaiz A4 (atau lain-lain kertas dengan persetujuan pensyarah berkaitan).

3. Write your details as follows in the upper left corner for each answer sheet: /
   Tulis butiran anda sepertimana berikut di penjuru atas kiri bagi setiap kertas jawapan:
   - *i.* Student Full Name / *Nama Penuh Pelajar*
   - ii. Identification Card (I/C) No. / *No. Kad Pengenalan*
   - iii. Class Section / *Seksyen Kelas*
   - iv. Course Code / *Kod Kursus*
   - v. Course Name / *Nama Kursus*
   - vi. Lecturer Name / *Nama Pensyarah*

4. Each answer sheet must have a page number written at the bottom right corner. /
   Setiap helai kertas jawapan mesti ditulis nombor muka surat di penjuru bawah kanan.

5. Answers should be **neat and clear in handwritten form**. /
   Jawapan hendaklah ditulis tangan, kemas dan jelas.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO /
JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU

This examination paper consists of **9** printed pages including front page
*Kertas soalan ini mengandungi **9** muka surat termasuk kulit hadapan*

PART A/ *BAHAGIAN A*

This part contains of **SIX (6)** questions. Answer **ALL** question in the answer sheet.

*Bahagian ini mempunyai **ENAM (6)** soalan. Jawab **SEMUA** soalan di dalam buku jawapan.*

QUESTION 1/ *SOALAN 1*

a) Differentiate, $\frac{dy}{dx}$ for the function below:

Bezakan, $\frac{dy}{dx}$ bagi fungsi-fungsi di bawah:

i. $y = 5x^2 + \frac{1}{x^4} - 2lnx + 5$

ii. $y = e^{x^2-3}$

(5 marks/ *markah*)

b) Find the equation of the tangent line and normal line for the curve $y = x^2 - 10$ at the point (3,-1)

Dapatkan persamaan garis tangen dan garis normal untuk lengkung $y = x^2 - 10$ pada titik (3,-1)

(7 marks/ *markah*)

QUESTION 2/ *SOALAN 2*

a) Integrate the following functions:
Kamirkan fungsi-fungsi berikut:

i) $\int \frac{1}{x} + 4x^2 - \sin x + 2 \, dx$

ii) $\int_0^1 (3 + 2x)^4 \, dx$ by substitutions method.
dengan kaedah gantian.

(5 marks/ *markah*)

b) Find the area of the region bounded by the curve $y = 3x - x^2$ and $y = x$ the line as shown in figure 1.
Dapatkan luas rantau yang dibatasi oleh lengkungan $y = 3x - x^2$ *dan garis* $y = x$ *yang ditunjukkan rajah 1.*

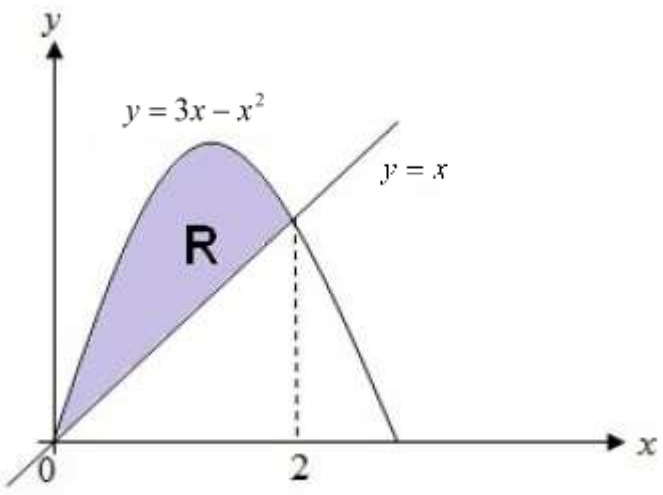

Figure 1 / Rajah 1

(5 marks/ *markah*)

QUESTION 3/ *SOALAN 3*

a) Use the method of separation of variables to solve the equation:

Gunakan kaedah pemisahan pemboleh ubah untuk menyelesaikan persamaan:

$$\frac{dy}{dx} = \frac{x^2 - 2x}{2y + y^2}$$

(4 marks/ *markah*)

QUESTION 4/ *SOALAN 4*

a) Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:

Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y''+y'-12y = 12x^2 + 2$$

(6 marks/ *markah*)

b) Find the Laplace Transforms of

Dapatkan Jelmaan Laplace berikut

$$L\{e^{3t} - e^{3t}\, sin\, 3\, t - \cos 5t\}$$

(3 marks/ *markah*)

c) Use the method of Laplace transforms to solve the initial value problem:

Guna kaedah jelmaan Laplace untuk menyelesaikan masalah nilai awal:

$$y'' - y' - 12y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

(7 marks/ *markah*)

QUESTION 5/ *SOALAN 5*

a) If $z = x^2 + y^2 + xy,\ x = \ln t$, and $y = e^t$, use the chain rule to find $\frac{\partial z}{\partial t}$.

Jika $z = x^2 + y^2 + xy,\ x = \ln t$, dan $y = e^t$, *gunakan aturan rantaian untuk mendapatkan* $\frac{\partial z}{\partial t}$

(4 marks/ *markah*)

b) Find the maximum, minimum and saddle points of the function:

Dapatkan titik maksimum, titik minimun dan titik pelana bagi fungsi:

$$f(x, y) = x^2 - 3xy + y^3$$

(6 marks/ *markah*)

QUESTION 6/ *SOALAN 6*

a) Sketch the region of integration and reverse the order of integration:

Lakarkan rantau kamiran dan tukarkan tertib kamiran:

$$\int_0^1 \int_0^y f(x, y)\ dxdy$$

(4 marks/ *markah*)

b) Use polar coordinates to evaluate the integral $\iint_R (x + y)\, dA$

where R is the region in the semicircle between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$

Gunakan kamiran kutub untuk menilaikan $\iint_R xy\, dA$ *, dengan* R *ialah rantau dalam separuh bulatan di antara bulatan* $x^2 + y^2 = 1$ *dan* $x^2 + y^2 = 16$.

(4 marks/ *markah*)

[60 marks/ *markah*]

END OF QUESTION PAPER/ *KERTAS SOALAN TAMAT*

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$$\cos^2 x + \sin^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

Hyperbolic Identities

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh 2x = 2\cosh^2 x - 1$$

$$\cosh 2x = 1 + 2\sinh^2 x$$

Derivatives Formulas

First Principle: $f'(x) = \lim_{\partial x \to 0} \frac{f(x+\partial x) - f(x)}{\partial x}$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cos ec\, x = -\cos ec\, x \cot x$$

$$\frac{d}{dx}\cot x = -\cos ec^2 x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a} = \frac{1}{x\ln a}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad , \qquad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \cos ec \, x \cot x \, dx = -\cos ec \; x + c$$

$$\int \cos ec^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \cos ec \; x \, dx = \ln|\cos ec \; x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c=Ae^{m_1x}+Be^{m_2x}$
2. real and equal: $m_1=m_2$	$y_c=Ae^{mx}+Bxe^{mx}$
3. complex numbers: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y_c=e^{\alpha x}\left(A\cos\beta x+B\sin\beta x\right)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1, m_2	y_p
$A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$m_1\neq 0$ and $m_2\neq 0$	$B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0$
	$m_1=0$ or $m_2=0$	$\left(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0\right)x$
$Ke^{\alpha x}$	$m_1\neq\alpha$ and $m_2\neq\alpha$	$Be^{\alpha x}$
	$m_1=\alpha$ or $m_2=\alpha$	$Bxe^{\alpha x}$
	$m_1=\alpha$ and $m_2=\alpha$	$Bx^2e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1\neq\beta i$ and $m_2\neq\beta i$	$B_1\cos\beta x+B_2\sin\beta x$
	$m_1=\beta i$ or $m_2=\beta i$	$\left(B_1\cos\beta x+B_2\sin\beta x\right)x$

Table of Laplace Transform $L\{f(t)\} = F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at} f(t)$	$F(s-a)$
8	$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at} \cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at} \sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at} \cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t \sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t \cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n = 1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y'(t)$	$sY(s) - y(0)$ with $Y(s) = L\{y\}$
19	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$