KOLEJ YAYASAN PELAJARAN JOHOR
FINAL EXAMINATION

COURSE NAME	:	**ENGINEERING MATHEMATICS III**
COURSE CODE	:	**MAT2033**
SESSION	:	**DECEMBER 2022**
DURATION	:	**3 HOURS**

INSTRUCTION TO CANDIDATES /
ARAHAN KEPADA CALON

1. This examination paper consists of **ONE (1)** part : / PART A (60 Marks) /
Kertas soalan ini mengandungi ***SATU (1)*** *bahagian:* *BAHAGIAN A (60 Markah)*

2. Candidates are not allowed to bring any material to the examination room except with the permission from the invigilator. The formula was attached at the back question paper. /
Calon tidak dibenarkan untuk membawa sebarang bahan/nota ke bilik peperiksaan tanpa arahan/kebenaran daripada pengawas. Rumus dilampirkan di belakang kertas soalan peperiksaan.

3. Please check to make sure that this examination pack consists of: /
Pastikan kertas soalan peperiksaan ini mengandungi:
   i. Question Paper /
   Kertas Soalan.
   ii. Answering Booklet /
   Buku Jawapan.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO /
JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU

This examination paper consists of **9** printed pages including front page
Kertas soalan ini mengandungi ***9*** *muka surat termasuk kulit hadapan*

This part contains of **SIX (6)** questions. Answer **ALL** questions in the Answering Booklet.
*Bahagian ini mempunyai **ENAM (6)** soalan. Jawab **SEMUA** soalan di dalam Buku Jawapan.*

QUESTION 1/ *SOALAN 1*

a) Differentiate, $\frac{dy}{dx}$ for the function below:

Bezakan, $\frac{dy}{dx}$ *bagi fungsi-fungsi di bawah:*

i. $y^2 - 3y + x^2 = 2x$ (by using Implicit Functions/ *menggunakan Fungsi Tersirat*)

ii. $x = sin(t + 3)$ and / *dan* $y = t^3$ (by using Parametric Functions/ *menggunakan Fungsi Berparameter*)

(7 marks / *markah*)

b) Find the equation of the tangent line and normal line for the curve $y = 2x^2 + x - 3$ at the point (1,0).
Dapatkan persamaan garis tangen dan garis normal untuk lengkung $y = 2x^2 + x - 3$ *pada titik* (1,0).

(5 marks / *markah*)

QUESTION 2/ *SOALAN 2*

a) Integrate the following functions:
Kamirkan fungsi-fungsi berikut:

i) $$\int 6\sqrt{x} - e^{x} + \frac{2}{x} + \sin x \, dx$$

ii) $\int_0^2 (3x^2)(x^3+1)^3 dx$ by substitutions method.
dengan kaedah gantian.

(5 marks / *markah*)

b) Find the area of the region bounded by the curve $y = x^2 + 2$ and $y = -x + 14$ the line as shown in figure 1.
Dapatkan luas rantau yang dibatasi oleh lengkungan $y = x^2 + 2$ dan garis $y = -x + 14$ yang ditunjukkan rajah 1.

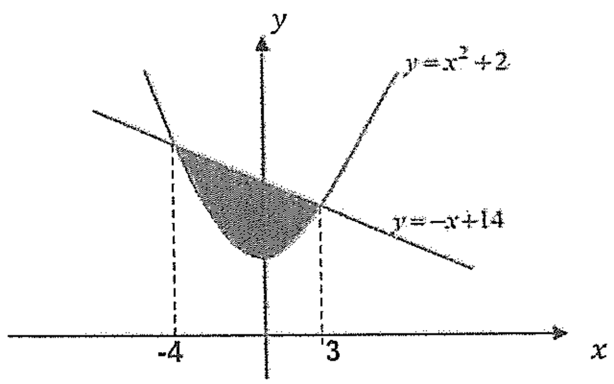

Figure 1 / *Rajah 1*

(5 marks / *markah*)

QUESTION 3/ *SOALAN 3*

Use the method of separation of variables to solve the equation:

Gunakan kaedah pemisahan pemboleh ubah untuk menyelesaikan persamaan:

$$e^{4x}\frac{dy}{dx} = 2y^3, \qquad y(0) = 1$$

(4 marks / *markah*)

QUESTION 4/ *SOALAN 4*

a) Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:

Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y'' + 4y' = 2x - 5$$

(6 marks / *markah*)

b) Find the Laplace Transforms of:

Dapatkan Jelmaan Laplace berikut:

$$L\{2t^4 - e^{3t}\cos 3\,t - t^2e^{3t}\}$$

(3 marks / *markah*)

c) Use the method of Laplace transforms to solve the initial value problem:

Guna kaedah jelmaan Laplace untuk menyelesaikan masalah nilai awal:

$$y'' + 4y' + 13y = 0, \quad y(0) = 2, \quad y'(0) = -7$$

(7 marks / *markah*)

QUESTION 5/ *SOALAN 5*

a) If $z = x^2y^3$, $x = 2t^3$, and $y = 3t^2$, use the chain rule to find $\frac{\partial z}{\partial t}$.

Jika $z = x^2y^3$, $x = 2t^3$ *dan* $y = 3t^2$ *gunakan aturan rantaian untuk mendapatkan* $\frac{\partial z}{\partial t}$.

(4 marks / *markah*)

b) Find the maximum, minimum and saddle points of the function:

Dapatkan titik maksimum, titik minimun dan titik pelana bagi fungsi:

$$f(x,y) = 2x^3 - 6xy - 3y^2$$

(6 marks / *markah*)

QUESTION 6/ *SOALAN 6*

a) Sketch the region of integration and reverse the order of integration:

Lakarkan rantau kamiran dan tukarkan tertib kamiran:

$$\int_0^2 \int_{y/2}^1 f(x,y)\; dxdy$$

(4 marks / *markah*)

b) Use polar coordinates to evaluate the integral

Gunakan kamiran kutub untuk menilaikan

$$\iint_R (x+y)\, dA$$

where R is the region in the first quadrant between the circles

dengan R ialah rantau dalam sukuan pertama di antara bulatan

$$x^2 + y^2 = 4 \text{ and / dan } x^2 + y^2 = 25.$$

(4 marks / *markah*)

[60 MARKS / *MARKAH*]

END OF QUESTION PAPER/ *KERTAS SOALAN TAMAT*

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Hyperbolic Identities

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\cosh^2 x - 1$

$\cosh 2x = 1 + 2\sinh^2 x$

Derivatives Formulas

First Principle: $f'(x) = \lim\limits_{\partial x \to 0} \dfrac{f(x+\partial x) - f(x)}{\partial x}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}x^n = nx^{n-1}$

$\dfrac{d}{dx}e^x = e^x$

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$

$\dfrac{d}{dx}\sec x = \sec x \tan x$

$\dfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\dfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x\ln a}$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, where $|x| < 1$

$\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$, where $|x| < 1$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c=Ae^{m_1x}+Be^{m_2x}$
2. real and equal: $m_1=m_2$	$y_c=Ae^{mx}+Bxe^{mx}$
3. complex numbers: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y_c=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1,m_2	y_p
$A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$m_1\neq 0$ and $m_2\neq 0$	$B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0$
	$m_1=0$ or $m_2=0$	$(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)x$
$Ke^{\alpha x}$	$m_1\neq\alpha$ and $m_2\neq\alpha$	$Be^{\alpha x}$
	$m_1=\alpha$ or $m_2=\alpha$	$Bxe^{\alpha x}$
	$m_1=\alpha$ and $m_2=\alpha$	$Bx^2e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1\neq\beta i$ and $m_2\neq\beta i$	$B_1\cos\beta x+B_2\sin\beta x$
	$m_1=\beta i$ or $m_2=\beta i$	$(B_1\cos\beta x+B_2\sin\beta x)x$

Table of Laplace Transform $L\{f(t)\} = F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at}f(t)$	$F(s-a)$
8	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at}\cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at}\cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t\sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t\cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n=1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y'(t)$	$sY(s)-y(0)$ with $Y(s)=L\{y\}$
19	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$