KOLEJ YAYASAN PELAJARAN JOHOR

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FINAL EXAMINATION

COURSE NAME	:	**ENGINEERING MATHEMATICS 3**
COURSE CODE	:	**MAT2033**
EXAMINATION	:	**OCTOBER 2019**
DURATION	:	**3 HOURS**

INSTRUCTION TO CANDIDATES /
ARAHAN KEPADA CALON

1. Answer **ALL** Question in the answer book
   *Jawab **SEMUA** soalan di dalam buku jawapan*

2. Candidates are not allowed to bring any material to examination room except with the permission from the invigilator. The formula was attached at the back question paper.
   Calon tidak dibenarkan untuk membawa sebarang bahan/nota ke bilik peperiksaan tanpa arahan / kebenaran daripada pengawas. Rumus dilampirkan di belakang kertas soalan peperiksaan.

3. Please check to make sure that this examination pack consist of: /
   Pastikan kertas soalan peperiksaan ini mengandungi:
   i. Question Paper/
      Kertas Soalan
   ii. Answer Booklet /
      Buku Jawapan

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO /
JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU

*This examination paper consists of **9** printed pages including front page*
*Kertas soalan ini mengandungi **9** muka surat termasuk kulit hadapan.*

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET

*Jawab **SEMUA** soalan di dalam buku jawapan*

QUESTON 1

a) Differentiate, $\frac{dy}{dx}$ for the function below:

Bezakan, $\frac{dy}{dx}$ *bagi fungsi-fungsi di bawah:*

i. $y = \frac{1}{2}x^4 + \sqrt{x} - 6$ (2 marks)

ii. $y = \ln\left(\sqrt{x} - 8\right)$ (3 marks)

b) Find the equation of the tangent line and normal line for the curve $x = 2\sqrt{t}$ and $y = 4t^2$ at the point where $t = 1$.

Dapatkan persamaan garis tangen dan garis normal untuk lengkung $x = 2\sqrt{t}$ *dan* $y = 4t^2$ *pada titik di mana* $t = 1$. (7 marks)

12 marks

QUESTON 2

a) Integrate the following functions:

Kamirkan fungsi-fungsi berikut:

i) $\int 3x^5 - 2x - \sqrt{8}\ dx$ (2 marks)

ii) $\int (7x+5)^5\, dx$ by substitutions method.

dengan kaedah gantian. (3 marks)

b) Find the area of the region bounded by the curve $y = x^2 - 9$ and the line $y = 3 - x$ as shown in figure 1.
Dapatkan luas rantau yang dibatasi oleh lengkungan $y = 5 - x^2$ *dan garis* $y = 3 - x$ *yang ditunjukkan rajah 1*

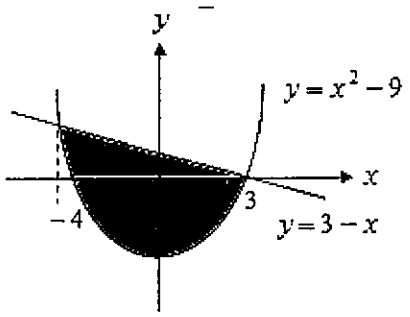

Figure 1

(4 marks)

9 marks

QUESTON 3

a) Use the method of separation of variables to solve the equation:
Gunkan kaedah pemisahan pemboleh ubah untuk menyelesaikan persamaan:

(5 marks)

$$\frac{dy}{dx} = \frac{x \sin x^2}{3y}$$

b) Determine the integrating factor and hence solve the differential equation.
Tentukan faktor pengamir dan selesaikan persamaan terbitan.

$$\frac{dy}{dx} + 3y = 2e^{5x}$$

(5 marks)

10 marks

QUESTON 4

a) Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:
Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y'' + 2y' = 2x$$ (7 marks)

b) Find the Laplace Transforms of :
Dapatkan jelmaan Laplace berikut:

i) $f(t) = 2t^4 - e^{3t}\cos 3t - t^2 e^{3t}$ (3 marks)

[10 marks]

QUESTON 5

a) If $z = 2xy + y^2$, where $x = u + v$ and $y = uv$, use the chain rule to f

$\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

Jika $z = 2xy + y^2$, $x = u + v$ *dan* $y = uv$, *gunakan aturan rantaian untuk mendapatkan* $\frac{\partial z}{\partial u}$ *dan* $\frac{\partial z}{\partial v}$ (5 marks)

b) Find the maximum, minimum and saddle points of the function:
Dapatkan titik maksimum, titik minimun dan titik pelana bagi fungsi: (5 marks)

$$f(x, y) = x^3 - xy + y^3$$

[10 marks]

QUESTON 6

a) Sketch the region of integration and reverse the order of integration:

Lakarkan rantau kamiran dan tukarkan tertib kamiran: (4 marks)

$$\int_0^2 \int_{2y}^4 f(x,y)\,dxdy$$

b) Use polar coordinates to evaluate the integral $\iint_R \left(\sqrt{x^2+y^2}\right) dA$ where R is the region enclosed by the circle $x^2+y^2=25$.

Gunakan kamiran kutub untuk menilaikan $\iint_R \left(\sqrt{x^2+y^2}\right) dA$, *dengan* R *ialah rantau dalam sukuan pertama di antara bulatan* $x^2+y^2=25$ (5 marks)

9 marks

END OF QUESTION PAPER / KERTAS SOALAN TAMAT

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Hyperbolic Identities

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\cosh^2 x - 1$

$\cosh 2x = 1 + 2\sinh^2 x$

Derivatives Formulas

First Principle: $f'(x) = \lim_{\partial x \to 0} \dfrac{f(x+\partial x) - f(x)}{\partial x}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}x^n = nx^{n-1}$

$\dfrac{d}{dx}e^x = e^x$

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$

$\dfrac{d}{dx}\sec x = \sec x \tan x$

$\dfrac{d}{dx}\cos ec\, x = -\cos ec\, x \cot x$

$\dfrac{d}{dx}\cot x = -\cos ec^2 x$

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x\ln a}$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$

$\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}, \quad where\ |x| < 1$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \cos ec \, x \cot x \, dx = -\cos ec \, x + c$$

$$\int \cos ec^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \cos ec \, x \, dx = \ln|\cos ec \, x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c=Ae^{m_1x}+Be^{m_2x}$
2. real and equal: $m_1=m_2$	$y_c=Ae^{mx}+Bxe^{mx}$
3. complex numbers: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y_c=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1, m_2	y_p
$A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$m_1\neq 0$ and $m_2\neq 0$	$B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0$
	$m_1=0$ or $m_2=0$	$(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)x$
$Ke^{\alpha x}$	$m_1\neq\alpha$ and $m_2\neq\alpha$	$Be^{\alpha x}$
	$m_1=\alpha$ or $m_2=\alpha$	$Bxe^{\alpha x}$
	$m_1=\alpha$ and $m_2=\alpha$	$Bx^2e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1\neq\beta i$ and $m_2\neq\beta i$	$B_1\cos\beta x+B_2\sin\beta x$
	$m_1=\beta i$ or $m_2=\beta i$	$(B_1\cos\beta x+B_2\sin\beta x)x$

Table of Laplace Transform $L\{f(t)\}=F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at}f(t)$	$F(s-a)$
8	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at}\cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at}\cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t\sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t\cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n=1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y(t)$	$Y(s)$
19	$y'(t)$	$sY(s)-y(0)$
20	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$