KOLEJ YAYASAN PELAJARAN JOHOR
FINAL EXAMINATION

COURSE NAME	:	**ENGINEERING MATHEMATICS 3**
COURSE CODE	:	**MAT2033**
EXAMINATION	:	**OCTOBER 2018**
DURATION	:	**3 HOURS**

INSTRUCTION TO CANDIDATES

1. Answer **ALL** Question in the answer booklet
2. Candidates are not allowed to bring any material to examination room except with the permission from the invigilator.
3. Please check to make sure that this examination pack consist of:
   i. Question Paper
   ii. Answer Booklet

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

*This examination paper consists of **9** printed pages including front page*

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET

Jawab SEMUA soalan di dalam buku jawapan

QUESTION 1

a) Differentiate, $\frac{dy}{dx}$ for the function below:

Bezakan fungsi-fungsi berikut terhadap $\frac{dy}{dx}$:

(i) $y = 5x^4 - \frac{2}{3}x + \sqrt{x}$

(ii) $y = 2kos\left(7x^2 + x\right)$

[4 Marks]

b) Find the equation of the tangent line to the curve $x = t$ and $y = \frac{1}{t}$ at the point where $t = -1$.

Dapatkan persamaan garis tangen untuk lengkung $x = t$ *dan* $y = \frac{1}{t}$ *pada titik di mana* $t = -1$.

[6 Marks]

QUESTION 2

(a) Integrate the following functions:

Kamirkan fungsi-fungsi berikut:

(i) $\int \left(12x^2 - 3x\right)dx$

(ii) $\int_0^2 (2x)\left(x^2 + 1\right)^3 dx$ by substitutions method.

dengan kaedah gantian.

[5 Marks]

(b) Find the area of the region bounded by the curve $y = x^2 - 9$ and the line $y = 3 - x$ as shown in figure 1.

Dapatkan luas rantau yang dibatasi oleh lengkungan $y = x^2 - 9$ *dan garis* $y = 3 - x$ *berikut seperti rajah 1.*

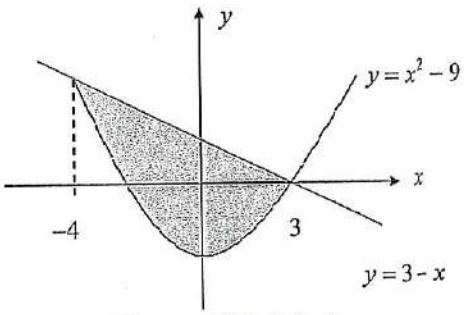

Figure 1/*Rajah 1*

[5 Marks]

QUESTION 3

(a) Use the method of separation of variables to solve the equation:

Guna kaedah pemisahan pembolehubah untuk menyelesaikan persamaan:

$$\frac{dy}{dx} = y \sin 2x$$

[5 Marks]

(b) Determine the integrating factor and hence solve the differential equation.

Tentukan faktor pengamir dan selesaikan persamaan terbitan.

$$\frac{dy}{dx} + \frac{y}{x} = x$$

[5 Marks]

QUESTION 4

a) Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:

Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y'' + y' - 6y = 2x$$

[6 Marks]

b) Find the Laplace Transforms of

Dapatkan Jelmaan Laplace bagi

(i) $f(t) = e^{2t}(\sinh 2t)$ (ii) $f(t) = e^{-3t} + t\cos 5t$

[4 Marks]

QUESTION 5

a) If $z = x^2 + y^2$, where $x = u + v$ and $y = u - v$, use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

Jika $z = x^2 + y^2$, *dengan* $x = u + v$ *dan* $y = u - v$, *gunakan aturan rantaian untuk mendapatkan* $\frac{\partial z}{\partial u}$ *dan* $\frac{\partial z}{\partial v}$.

[5 Marks]

b) Find the maximum, minimum and saddle points of the function:

Dapatkan titik maksimum, titik minimum dan titik pelana bagi fungsi:

$$f(x, y) = x^2 - 6x + 2y^2 + 4y - 2$$

[5 Marks]

QUESTION 6

a) Sketch the region of integration and and reverse the order of integration:
Lakarkan rantau kamiran dan tukarkan tertib kamiran:

$$\int_0^3 \int_1^{4-x} f(x, y)\,dy\,dx$$

[4 Marks]

b) Use polar coordinates to evaluate the integral $\iint_R 4y\,dA$, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 36$
Gunakan kamiran kutub untuk menilaikan $\iint_R 4y\,dA$, *dengan* R *ialah rantau tertutup dalam sukuan pertama bulatan* $x^2 + y^2 = 36$

[6 Marks]

END OF QUESTION PAPER

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Hyperbolic Identities

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\cosh^2 x - 1$

$\cosh 2x = 1 + 2\sinh^2 x$

Derivatives Formulas

First Principle: $f'(x) = \lim_{\partial x \to 0} \dfrac{f(x+\partial x) - f(x)}{\partial x}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}x^n = nx^{n-1}$

$\dfrac{d}{dx}e^x = e^x$

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$

$\dfrac{d}{dx}\sec x = \sec x \tan x$

$\dfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\dfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x\ln a}$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, where $|x| < 1$

$\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$, where $|x| < 1$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c = Ae^{m_1x}+Be^{m_2x}$
2. real and equal: $m_1=m_2$	$y_c = Ae^{mx}+Bxe^{mx}$
3. complex numbers: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y_c = e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1,m_2	y_p
$A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$m_1\neq 0$ and $m_2\neq 0$	$B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0$
	$m_1=0$ or $m_2=0$	$(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)x$
$Ke^{\alpha x}$	$m_1\neq\alpha$ and $m_2\neq\alpha$	$Be^{\alpha x}$
	$m_1=\alpha$ or $m_2=\alpha$	$Bxe^{\alpha x}$
	$m_1=\alpha$ and $m_2=\alpha$	$Bx^2e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1\neq\beta i$ and $m_2\neq\beta i$	$B_1\cos\beta x+B_2\sin\beta x$
	$m_1=\beta i$ or $m_2=\beta i$	$(B_1\cos\beta x+B_2\sin\beta x)x$

Table of Laplace Transform $L\{f(t)\} = F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at} f(t)$	$F(s-a)$
8	$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at} \cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at} \sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at} \cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t \sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t \cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n = 1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y(t)$	$Y(s)$
19	$y'(t)$	$sY(s) - y(0)$
20	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$