KOLEJ YAYASAN PELAJARAN JOHOR
FINAL EXAMINATION

COURSE NAME	:	**ENGINEERING MATHEMATICS 3**
COURSE CODE	:	**MAT2033**
EXAMINATION	:	**APRIL 2018**
DURATION	:	**3 HOURS**

INSTRUCTION TO CANDIDATES

1. Answer **ALL** Question in the answer booklet

2. Candidates are not allowed to bring any material to examination room except with the permission from the invigilator.

3. Please check to make sure that this examination pack consist of:
   i. Question Paper
   ii. Answer Booklet

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*This examination paper consists of **9** printed pages including front page*

ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET

Jawab SEMUA soalan di dalam buku jawapan

QUESTION 1

a) Differentiate, $\frac{dy}{dx}$ for the function below:

Bezakan, $\frac{dy}{dx}$ bagi fungsi-fungsi di bawah:

(i) $y = x^5 - \sqrt{x} + \frac{2}{x^3}$

(ii) $y = 3\sin(2x^2 - 5)$

[4 Marks]

b) Find the equation of the tangent line to the curve $x = \ln t$ and $y = 2t^2$ at the point where $t = 1$.

Dapatkan persamaan garis tangen untuk lengkung $x = \ln t$ dan $y = 2t^2$ pada titik di mana $t = 1$.

[6 Marks]

QUESTION 2

(a) Integrate the following functions:

Kamirkan fungsi-fungsi berikut:

(i) $\int_1^3 8x^3 dx$

(ii) $\int (4x+3)^5 dx$ by substitutions method.

dengan kaedah gantian.

[4 Marks]

(b) Use the method of separation of variables to solve the equation:

Guna kaedah pemisahan pembolehubah untuk menyelesaikan persamaan:

$$\frac{dy}{dx} = \frac{1}{xy + y}$$

[6 Marks]

QUESTION 3

Use the method of undetermined coefficients to solve the nonhomogeneous differential equation:

Gunakan kaedah pekali tak ditentukan untuk menyelesaikan persamaan tak homogen:

$$y'' - 10y' + 25y = 30x + 3$$

[10 Marks]

QUESTION 4

a) Evaluate:

Nilaikan:

$$L\{3 - 2e^{5t} + 5\sin 2t\}$$

[3 Marks]

b) Solve the equation by using first shift theorem.

Selesaikan persamaan dengan menggunakan teorem anjakan pertama.

$$L\{e^{5t}\cosh 2t\}$$

[5 Marks]

c) Find the inverse Laplace transform of the function.

Dapatkan penjelmaan Laplace songsang untuk fungsi.

$$L^{-1}\left\{\frac{1}{s^2 + 9}\right\}$$

[2 Marks]

QUESTION 5

a) If $f(x,y)=x^2+xy+y^2$, where $x=u+v$ and $y=uv$, use the chain rule to find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$.

Jika $f(x,y)=x^2+xy+y^2$, *dengan* $x=u+v$ *dan* $y=uv$, *gunakan aturan rantaian untuk mendapatkan* $\frac{\partial f}{\partial u}$ *dan* $\frac{\partial f}{\partial v}$.

[5 Marks]

b) Find the maximum, minimum and saddle points of the function:

Dapatkan titik maksimum, titik minimum dan titik pelana bagi fungsi:

$$f(x,y)=x^2-xy+y^2$$

[5 Marks]

QUESTION 6

a) Sketch the region of integration and reverse the order of integration:

Lakarkan rantau kamiran dan tukarkan tertib kamiran:

$$\int_0^2\int_{2y}^4 f(x,y)\,dxdy$$

[5 Marks]

b) Evaluate the double integral $\iint_R (4-x-y)dA$ where R is rectangle defined by $0\le x\le 1$ and $0\le y\le 2$

Nilaikan kamiran ganda dua $\iint_R (4-x-y)dA$ *dimana* R *ialah segiempat tepat yang ditakrifkan oleh* $0\le x\le 1$ *dan* $0\le y\le 2$

[5 Marks]

END OF QUESTION PAPER

LIST OF FORMULA

SENARAI RUMUS

Basic Identities

Trigonometric Identities

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Hyperbolic Identities

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\cosh^2 x - 1$

$\cosh 2x = 1 + 2\sinh^2 x$

Derivatives Formulas

First Principle: $f'(x) = \lim_{\partial x \to 0} \dfrac{f(x + \partial x) - f(x)}{\partial x}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}x^n = nx^{n-1}$

$\dfrac{d}{dx}e^x = e^x$

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$

$\dfrac{d}{dx}\sec x = \sec x \tan x$

$\dfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\dfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x\ln a}$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$, *where* $|x| < 1$

$\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1 - x^2}}$, *where* $|x| < 1$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$

Integrals Formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \tan x \, dx = \ln|\sec x| + c$$

$$\int \cot x \, dx = \ln|\sin x| + c$$

$$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + c$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$$

The method of undetermined coefficients

Solution of homogeneous equation: $ay''+by'+cy=0$

Auxiliary equation: $am^2+bm+c=0$

Roots of $am^2+bm+c=0$	General Solution, y_c
1. real and different: m_1 and m_2	$y_c = Ae^{m_1 x} + Be^{m_2 x}$
2. real and equal: $m_1 = m_2$	$y_c = Ae^{mx} + Bxe^{mx}$
3. complex numbers: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular integrals of inhomogeneous equation: $ay''+by'+cy=f(x)$

$f(x)$	Roots of auxiliary equation: m_1, m_2	y_p
$A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$m_1 \neq 0$ and $m_2 \neq 0$	$B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0$
	$m_1 = 0$ or $m_2 = 0$	$(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)x$
$Ke^{\alpha x}$	$m_1 \neq \alpha$ and $m_2 \neq \alpha$	$Be^{\alpha x}$
	$m_1 = \alpha$ or $m_2 = \alpha$	$Bxe^{\alpha x}$
	$m_1 = \alpha$ and $m_2 = \alpha$	$Bx^2 e^{\alpha x}$
$K\cos\beta x$ or $K\sin\beta x$	$m_1 \neq \beta i$ and $m_2 \neq \beta i$	$B_1\cos\beta x + B_2\sin\beta x$
	$m_1 = \beta i$ or $m_2 = \beta i$	$(B_1\cos\beta x + B_2\sin\beta x)x$

Table of Laplace Transform $L\{f(t)\} = F(s)$

	$f(t)$	$F(s)$
1	a	$\frac{a}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	$\sin at$	$\frac{a}{s^2+a^2}$
4	$\cos at$	$\frac{s}{s^2+a^2}$
5	$\sinh at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$e^{at}f(t)$	$F(s-a)$
8	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
9	$e^{at}\cos bt$	$\frac{(s-a)}{(s-a)^2+b^2}$
10	$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
11	$e^{at}\cosh bt$	$\frac{(s-a)}{(s-a)^2-b^2}$
12	$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
13	$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
14	$t\sinh at$	$\frac{2as}{(s^2-a^2)^2}$
15	$t\cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
16	$t^n, n=1,2,3...$	$\frac{n!}{s^{n+1}}$
17	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
18	$y(t)$	$Y(s)$
19	$y'(t)$	$sY(s)-y(0)$
20	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$